(25) (Fn Φ_n

xmigroup (

is (Belaukin

1973

M, $\alpha \in$ Aut

st. $\Phi_n^{(x)} = E$ (

Fn Hecke

$(\exists)\, t : \mathcal{H} \rightarrow$

st.

$\Psi_{[\Gamma \circ \Gamma]}^{(x)} =$

X

26

t is constructed

out of π

$$t(A) = \sum_{\theta \in A} \overline{\langle \pi(\theta)\xi, \xi \rangle}\, \theta$$

($\xi \in \ell^2(\Gamma)$ unit vector)

Step 2 Find essential spectrum of $\Psi_{[\Gamma \backslash \Gamma]}$ expressed like above

$$\Psi_{[\Gamma \backslash \Gamma]} \in \bigoplus_{r} C^*(\text{left conj, right conj, characteristic fcts}) x_r$$

$x_r \leftrightarrow E^{\perp}(\theta)$.

24 So one has to describe the image in Calkin algebra $B(\ell^2(\Gamma))/K(\ell^2(\Gamma))$ of $E_\Gamma^G C^*(L(G), R(G), \chi_G) E_\Gamma^0$ which is a sup of $\chi_\Gamma(C^*(G \ltimes G^{(0)}, \chi_{\Gamma^2 S}, 3))\chi_\Gamma$

Recall A.O. for $G = F_N$ or hyperbolic (OZ) or lattice in Lie Group (Skandalis)

$C^*(L(G), R(U))/K \cong C_{red}^*(G \rtimes G^{(0)})$

(28) Step 2
Identify essential states

Let $\Phi_0 : \mathbb{C}^*_r((6 \times 6^T) \rtimes X_r)$

characteristic fctns of cosets

defined by

$$\Phi_0\left(\sum_{g=(g_1,g_2)} g f_g\right) =$$

$$= \sum \chi(g_1, g_2) \frac{\int f_g \, d(\text{Haar})}{g_1\Gamma g_2 \cap \Gamma}$$

$$\chi(g_1, g_2) = \lim_{t \to \infty} \frac{\text{vol}(B_t \cap g_1 B_t g_2)}{\text{vol}(B_t)}$$

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All other essential states are of the form

$$X \to \underline{\Omega}(\Sigma^* \times \Sigma)$$

One computes

$$H \to \chi_\Gamma C^*(G \times C^T \times X_\Gamma)\chi_\Gamma$$

$$\downarrow$$

$$\Omega(\Sigma^* \times \Sigma)$$

is continuous □